Effects of dust charge fluctuations on current-driven dust-ion-acoustic waves

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The effects of fluctuating grain charge on current-driven dust-ion-acoustic modes are investigated in the presence of various physical parameters which are varied in order to describe the behavior of the mode in detail. Variation of the charge clearly influences the ion-acoustic mode by changing of its frequency and growth rate. We report here the existence of a dust charge fluctuation mode in the system, and it is shown that the mode is unstable for a negative charge on the dust per unit volume not exceeding some critical value.

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I. INTRODUCTION

According to the present knowledge of processes in dusty and multicomponent plasmas, there are two main differences that clearly distinguish these two systems, i.e., the dust particles are usually heavy compared to the standard plasma components, and there is a fluctuating charge residing on them. Both effects have been investigated extensively in the past decade.

The large difference in masses has been used in analytical studies in the past in two different limits. The first implies dusty particles so heavy that they can be taken as stationary for frequencies much exceeding the characteristic frequencies of the dusty plasma component, making in that way only a stationary neutralizing background for the perturbations propagating in the much lighter electron-ion component of the plasma. If, in the same limit, the dust density is nonuniform, this can cause an additional mode driven by the dust density gradient [1]. The investigation of nonlinear electrostatic [2] and electromagnetic [3,4] perturbations propagating in such a system reveals the possibility of formation of various types of vortical structure. The other limit, valid for very low frequencies that are of the order of, or less than, typical plasma frequencies of the dusty fluid, includes the dynamics of dust particles [5], and reveals the possibility for extremely low-frequency modes (of the order of hertz), and with wavelengths that are visible to the naked eye. Predicted by theory, these modes have been successfully detected in laboratory conditions [6].

The variation of the charge on dust particles introduces an additional temporal scale in a dusty plasma. Owing to its high-frequency nature (\sim of the order of megahertz), the charge variation, on one hand, may not be dynamically important for dust-acoustic waves (\sim few hertz), and yet it will influence the collective behavior of the plasma particles, on the other hand. Therefore, some of the very low-frequency processes in a dusty plasma are uninfluenced by the charge fluctuation, so that taking some average value of the charge is usually a good enough assumption [1–5,7]. However, the plasma modes with frequencies close to the average charging frequency of the grains, like the dust-ion-acoustic mode, will

be affected by the charge dynamics. Therefore, the presence of dust in a plasma introduces two widely separated and well defined time scales that can be studied separately.

In the present paper we investigate the effects of the fluctuation of the charge on dust particles on the current-driven dust-ion-acoustic (DIA) mode, first predicted in Ref. [8]. The physical interpretation of this mode is similar to that of the standard ion-acoustic (IA) mode, with a modification caused by the presence of a stationary background of charged dust grains. Both DIA and IA modes can be relatively easily produced in laboratory conditions by applying a constant steady state electric field. Recently [9], a detailed analysis of the DIA modes produced by a relative drift of electrons and ions as a result of a constant, externally applied electric field was performed for various values of the physical parameters describing the system. One conclusion is that the more dust in the system, the easier is DIA mode excitation in the system. The explanation of this effect follows from the fact that the bigger the number of dust particles (usually negatively charged by mobile electrons), the smaller the number of electrons required to neutralize the eventual space charge variation of ions, and, therefore, the system becomes more unstable. This, and other results from Ref. [9], are reexamined in the presence of a fluctuating charge on the dust particles, and the corresponding dispersion equation is solved and discussed numerically. Also, we report here the existence of a charge fluctuation (CF) mode which not only influences the DIA mode but also has a frequency almost matching that of the ion-acoustic mode for certain values of the large wavelength. This feature of the CF mode makes it an attractive candidate for resonant excitation of the ion-acoustic mode. Recently, a similar analysis of the instability of currentdriven DIA modes was performed in Ref. [10]. However, only the threshold for the instability of current-driven DIA modes was derived for the case of marginal stability, and discussed in detail. In the present analysis DIA and CF modes are studied numerically by varying various parameters that describe the system.

II. MODEL AND EQUATIONS

As in Ref. [9], we study a plasma consisting of ions, electrons, negatively charged dust grains, and neutrals. The effects of neutrals will appear in the corresponding equations via electron- and ion-neutral collision frequencies ν_e and ν_i ,

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respectively. In describing small electrostatic perturbations propagating in the system, we use the continuity and momentum equations for ions and electrons,

$$\frac{\partial n_i}{\partial t} + \nabla(n_i \vec{v}_i) = -\nu_{id}(n_i - n_{i0}), \qquad (1)$$

$$n_i m_i \left(\frac{\partial \vec{v}_i}{\partial t} + \vec{v}_i \cdot \nabla \vec{v}_i \right) = -\kappa T_i \vec{\nabla} n_i + e n_i \vec{E} - \nu_i n_i m_i \vec{v}_i, \quad (2)$$

$$\frac{\partial n_e}{\partial t} + \nabla (n_e \vec{v_e}) = -\nu_{ed} (n_e - n_{e0}), \qquad (3)$$

$$\vec{0} = \kappa T_e \vec{\nabla} n_e + e n_e \vec{E} + \nu_e n_e m_e \vec{v}_e \,. \tag{4}$$

Here the terms on the right-hand sides of Eqs. (1) and (3) are the source or sink terms caused by inelastic collisions with dust particles, and $\nu_{ed,id}$ are the electron-dust and ion-dust collision frequencies. In Eq. (4) the electron inertia on the left-hand side is neglected, but kept in the collision term, which can be done [9] since in the collision term it is of a lower order ($\sim m_e^{1/2}$).

Further, we use the charge conservation equation, which can be written as

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0, \qquad (5)$$

where

$$\rho = en_i - en_e - eZ_dn_d, \quad \vec{j} = en_i \vec{v}_i - en_e \vec{v}_e - eZ_dn_d \vec{v}_d.$$

Using Eqs. (1) and (3), from Eq. (5) we obtain the following charge fluctuation equation:

$$\frac{\partial Q}{\partial t} + \vec{v}_d \cdot \vec{\nabla} Q = \frac{e \nu_{ed}}{n_{d0}} \left[n_e - n_{e0} - \frac{\nu_{id}}{\nu_{ed}} (n_i - n_{i0}) \right], \quad (6)$$

where

$$\begin{split} \nu_{ed} &= P_c \Omega_c \frac{\tau + z}{1 + \tau + z}, \quad \nu_{id} = \sqrt{\frac{m_e}{m_i}} \tau \nu_{ed} \exp(z), \\ P_c &= \frac{a n_{d0} \kappa T_e}{n_{e0} e^2}, \\ \tau &= \frac{T_i}{T_e}, \quad z = \frac{Qe}{a \kappa T_e}, \quad \Omega_c &= \omega_{pi}^2 a (1 + \tau + z) / c_s \sqrt{2 \pi}, \\ c_s^2 &= \frac{\kappa T_i}{m_i}, \end{split}$$

a is the dust particle radius, and the other notations are standard. The value of the parameter P_c covers a rather broad range, especially in the case of astrophysical and space dusty plasmas. It is 10^{-4} for Saturn's *E* ring, 10 for Saturn's *G* ring, and 1 for Jupiter's ring [11]. In Eq. (6) $Q \equiv eZ_d$ denotes the absolute value of the charge on the dust particles, caused by inelastic collisions with electrons and ions, and generally it is not a constant. Its variation is usually a high-frequency (of the order of megahertz) process. As already pointed out a dusty plasma introduces two widely separated time scales, namely, the dust-acoustic and charge fluctuation scales, and for many practical situations and for low-frequency modes, the charge residing on the dust grains can be considered to have some average value [7] and its fluctuation can be completely ignored. The fluctuation of charge, however, is of importance for normal plasma modes whose frequencies are not so well separated from the charging frequency of the grains, and the fluctuating charge on dust particles can give rise to damping or growth of the mode in a reasonably short time, which is of the order of the period of a plasma wave oscillation.

In equilibrium, the charge on the dust particle is assumed negative (due to the usually higher mobility of electrons, compared to ions), and constant (due to the balance of the electron and ion currents $I_{e0,i0}$). However, in the presence of collective perturbations propagating through the system, which cause the space-time variation of physical quantities determining the amount of charge on the dust particles, the charge becomes space-time dependent, and therefore Eq. (6) is to be studied self-consistently with Eqs. (1)–(4). At the same time, in the present model, the heavy dust particles are assumed to be stationary, so that the dust dynamics is described only by the fluctuation of the charge, which is physically justified having in mind the difference of characteristic frequencies in the dust and in the electron-ion plasma component.

We assume a stationary equilibrium in the uniform plasma in the presence of a constant electric field E_0 . Equations (2) and (4), together with the plasma quasineutrality condition in equilibrium

$$n_{i0} = n_{e0} + Z_{d0} n_{d0}, \tag{7}$$

in that case yield

$$eE_0 = \nu_i m_i v_{i0} = -\nu_e m_e v_{e0}, \qquad (8)$$

$$n_{e0} = (1 - \delta) n_{i0} \,. \tag{9}$$

Here $\delta = n_{d0}Z_{d0}/(n_{e0} + n_{d0}Z_{d0})$ is a parameter representing the fraction of the total negative charge (dust+electrons) per unit volume attributed to dust grains only.

The electric field introduced above can be easily achieved in experimental conditions. In such situations it is responsible for the electron and ion currents, resulting, e.g., in current-driven ion-acoustic [12] or dust-ion-acoustic waves [9]. At this stage it is worth noting that the presence of an electric field in a dusty plasma can be of importance for space and astrophysical applications as well; namely, a proper analysis of the equilibrium of a gravitating (or selfgravitating) dusty plasma system requires the presence of a large scale static electric field, resulting from the inevitable charge separation caused by the difference of masses of plasma components. This effect in ordinary astrophysical plasma systems (i.e., for an electron-ion plasma) has been known for a long time (Pannekoek-Rosseland field) [13]. In

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the presence of charged dust, however, the situation should dramatically change; it can be shown that the ratio of dust to electron scale lengths $(L_d/L_e \sim m_d T_e/m_e T_d)$ is very large. For the electron scale length of a few kilometers, the dust density scale length becomes of the order of a few parsecs, implying the existence of a large scale electric field that in fact could explain the large scale flows in astrophysical systems.

For a perturbed plasma and for small perturbations of the form $\sim \exp(-i\omega t + ikx)$, Eqs. (1)–(6) are linearized yielding the following dispersion equation, which will be the basic one for further analysis:

$$(1-\delta)\left(1+\frac{iP_c\Omega_c}{\Omega_1}\right)\left[\Omega(\Omega+i\nu_i)-k^2c_s^2\right] -\left[k^2c_i^2-i\nu_e\frac{m_e}{m_i}(\Omega-ku_0)\right]\left(1+\frac{i\epsilon P_c\Omega_c}{\Omega_1}\right)=0.$$
(10)

Here the following notations and definitions for parameters are used:

$$\Omega = \omega - kv_{i0}, \quad \Omega_1 = \Omega + kv_{i0}, \quad u_0 = v_{e0} - v_{i0},$$
$$c_i^2 = \kappa T_e / m_i,$$
$$\epsilon \equiv \frac{v_{id}}{v_{ed}} = \tau \sqrt{m_e / m_i} \exp(z).$$

Equation (10) comprises several physical parameters like E_0 (i.e., v_{i0}, u_0), Ω_c , ϵ , δ , the pressure of neutrals etc. If $\epsilon = 1$, i.e., when the ions and electrons are colliding with the grains at the same rate, we should expect that the average charge on the grain will remain stationary and there should be no effect of the dust grains on the ion-acoustic waves. Indeed, for $\epsilon = 1$, we see from Eq. (10) that the charge fluctuation effect factors out and we have the usual collisional instability of the acoustic mode with

$$\gamma \sim \frac{1}{2} \frac{m_e}{m_i} \nu_e \left(1 + \frac{Z_{d0} n_{d0}}{n_{e0}} \right) \frac{u_0}{\sqrt{c_s^2 + (1 + Z_{d0} n_{d0} / n_{e0}) c_i^2}} - \frac{\nu_i}{2} - \frac{m_e}{2m_i} \left(1 + \frac{Z_{d0} n_{d0}}{n_{e0}} \right) \nu_e , \qquad (11)$$

which is similar to the result in Ref. [9]. Here, while deriving the above equation, we have assumed that $\gamma \ll \Omega_r$ and ignored the terms $\sim O(\gamma^2, \nu \gamma)$. Next, when $\epsilon \neq 1$, we first solve Eq. (11) perturbatively. Assuming again $\gamma \ll \Omega_r$ and ignoring terms $\sim O(\nu_{i,e}^2, \nu_{i,e}\gamma)$, we find that

$$\Omega_r^2 = k^2 \left[c_s^2 + \left(1 + \frac{Z_{d0} n_{d0}}{n_{e0}} \right) c_i^2 \right],$$

and the growth rate is given by

$$\gamma \sim \frac{1}{2} \frac{m_e}{m_i} \nu_e \left(1 + \frac{Z_{d0} n_{d0}}{n_{e0}} \right) \frac{u_0}{\sqrt{c_s^2 + (1 + Z_{d0} n_{d0} / n_{e0}) c_i^2}} - \frac{\nu_i}{2} - \frac{m_e}{2m_i} \left(1 + \frac{Z_{d0} n_{d0}}{n_{e0}} \right) \nu_e + \frac{P_c \Omega_c}{2} \times \left\{ \frac{\left[c_s^2 + \epsilon c_i^2 (1 + Z_{d0} n_{d0} / n_{e0}) \right]}{\left[c_s^2 + c_i^2 (1 + Z_{d0} n_{d0} / n_{e0}) \right]} - 1 \right\}.$$
(12)

This mode is excited by charge fluctuation on the grain. Therefore, we shall anticipate the existence of a charge fluctuation induced growing mode from our numerical analysis of Eq. (10). Note that for $\epsilon \ll 1$, i.e., when the electron is the prime cause of the charge fluctuation, the CF mode is very small. However, when $\epsilon > 1$, the mode may become important.

Next, we solve Eq. (10) numerically. For this purpose, we adopt the following numerical values for physical constants:

$$T_e = 2 \text{ eV}, \ T_i = 0.1 \text{ eV}, \ \nu_i = 7.35 \times 10^3 P \text{ (mtorr)},$$

 $\nu_e = 3.5 \times 10^5 P \text{ (mtorr)},$
 $u_0 \text{ (m/s)} = -5 \times 10^5 \frac{E_0}{P},$
 $c_{ia} = \sqrt{(\kappa T_e + \kappa T_i)/m_i} = 2.2 \times 10^3 \text{ m/s}.$

III. NUMERICAL RESULTS

In the case $\Omega_c = 0$ (no charge fluctuation) Eq. (10) yields two roots of the type $\Omega_r + \gamma_1$, $-\Omega_r + \gamma_2$, where $\gamma_1 > 0$ and $\gamma_2 < 0$. The unstable physically interesting root was investigated in Ref. [9].

Here, we first solve Eq. (10) in the ion reference frame for the real Ω_r and imaginary γ parts of the DIA mode frequency Ω , by varying the charging frequency Ω_c up to 5



FIG. 1. The nondimensional real $-\Omega_r$ (solid line) and imaginary γ part (dashed line) of the frequency Ω , in units of kc_{ia} , of the DIA wave driven by the current, as a function of the charge variation frequency Ω_c . The values of parameters are given in the text.



FIG. 2. The negative DIA frequency $-\Omega_r$ versus k, for $\epsilon = 3$ (solid line), $\epsilon = 0.3$ (dashed line), and in the absence of charge fluctuation (dotted line). Parameters are $\delta = 0.5$, $E_0 = 250$ V/m, $P_c = 0.9$, P = 10 mtorr, $\Omega_c = 10^5$ Hz.

 $\times 10^{6}$ Hz, and for $P_c = 0.9$, P = 10 mtorr, $\epsilon = 0.3$, $\delta = 0.5$, $E_0 = 250$ V/m, and k = 63 m⁻¹. The result is presented in Fig. 1. Both the frequencies Ω_r (the solid line) and γ (the dashed line) are in units of $kc_{ia} = 138.6 \times 10^3 \text{ s}^{-1}$. The increment γ has maximum value $\gamma_{max} \approx 0.93$ at $\Omega_c = 2$ $\times 10^5$ Hz. The horizontal line gives the value of $\gamma = 0.837$ in the absence of charge fluctuation. As a result of charge fluctuation the increment of the DIA mode is increased for a charging frequency not exceeding a critical value ≈ 6 $\times 10^5$ Hz. It can be shown that this critical value grows for bigger intensity of the applied field; for $E_0 = 100$ V/m it is $\approx 2.6 \times 10^5$ Hz, and for $E_0 = 500$ V/m it is 1.2×10^6 Hz. Above these values the increment decreases considerably. Whereas the enhancement of the DIA growth rate due to charge fluctuation is not significant, the suppression of the growth rate is rapid. Clearly then, when the grain charging frequency Ω_c is below a certain threshold value, the electric field in the vicinity of a fluctuating charge is in phase with the externally imposed field and this results in enhancement of the growth rate. However, when the fluctuation of the charge becomes too rapid, the repulsive field of the grain aligns itself opposite to the externally imposed field and this



FIG. 4. The negative DIA frequency $-\Omega_r/kc_{ia}$ versus the applied electric field E_0 . Here $k=63 \text{ m}^{-1}$, P=50 mtorr, $P_c=0.9$. The solid line is for $\epsilon=3$, the dashed line for $\epsilon=0.3$, and the dotted line is in the absence of charge fluctuations.

results in a rapid fall of the DIA growth rate. At the same time charge fluctuation introduces a certain inertia in the behavior of the DIA mode since its frequency becomes almost half of its value without the charge fluctuation.

In Fig. 2 is given the real part of the frequency Ω_r of the DIA mode versus wave number k. The solid line is for ϵ =3, the dotted line is for constant charge on the dust particles, and the dashed line is for $\epsilon = 0.3$. In the absence of charge variation (dotted line) two DIA modes (one stable and one unstable) are present in the system, with frequencies of opposite signs, corresponding to two possible directions of propagation. For $\epsilon = 0.3$, the same two DIA modes, albeit somewhat modified, are identified, along with a charge fluctuation induced mode. The existence of the third mode, namely, the charge fluctuation induced mode, was reported earlier in studies of DIA modes in the presence of charge variations [14]. The growth rates of the unstable DIA modes, γ , are presented in Fig. 3, normalized to the value at k =10³ m⁻¹. Here $\gamma_{(k=1000)} = 0.98 \times 10^5$ Hz (for $\epsilon = 3$), $\gamma_{(k=1000)} = 1.74 \times 10^5$ Hz (for $\epsilon = 0.3$), and $\gamma_{(k=1000)} = 1.54$ $\times 10^5$ Hz (in the absence of charge fluctuation). For $\epsilon = 3$ (the solid line), we find that around $k \approx 30 \text{ m}^{-1}$ the real part



FIG. 3. Normalized increment $\gamma/\gamma(k=1000)$ versus k for frequencies from Fig. 2.



FIG. 5. The increments $\gamma/|\Omega_r|$ versus E_0 for the frequencies from Fig. 4.



FIG. 6. Critical electric field E_c for the onset of the instability versus the amount of dust charge δ in the system. Dotted line is for the absence of charge fluctuation and the solid line for $\epsilon = 3$. Here $k = 63 \text{ m}^{-1}$, P = 100 mtorr, $P_c = 0.9$.

of the CF mode frequency becomes very close to the frequency of the DIA mode. The signature of this is seen in the growth rate of the DIA mode (Fig. 3), as after attaining a maximum (solid line) the DIA mode decays. The matching of the CF and DIA mode frequencies suggests that a DIA mode can be excited in a dusty plasma when the variation of the charge on the dust grains is predominantly carried by ions. Notice that $\epsilon = 3$ physically means that the variation of the charge on the dust grains in this limit is predominantly caused by ions [see Eq. (6) and the definition of ϵ], i.e., the electrons are already mostly attached to the negatively charged grains. As the presence of a negative charged grain in a positive ion background is tantamount to introducing an electric field, the excitation of the DIA mode by charge fluctuation is equivalent to its excitation by an applied electric field.

In Figs. 4 and 5, we present Ω_r and γ of the DIA mode as functions of the intensity of the applied electric field E_0 . The dotted lines in the figures correspond to the case without charge variation. Obviously the applied electric field has a destabilizing effect, and can be used as well for the excitation of DIA mode. The mode becomes unstable at $E_0 \approx$ 70 V/m, $E_0 \approx 230$ V/m, and $E_0 \approx 170$ V/m, for $\epsilon = 3$, ϵ



FIG. 8. Mode frequency $-\Omega_r/kc_{ia}$ versus δ . Solid line, for $\epsilon = 3$; dotted line, for the absence of charge fluctuation ($\Omega_c = 0$); dashed and dash-dotted lines for $\epsilon = 0.3$. The last one (dash-dotted) represents the dust charge fluctuation mode. The parameters are given in the text.

=0.3, and $\Omega_c = 0$, respectively. The curves for $\epsilon = 0.3$ (dashed) and $\epsilon = 3$ (solid) (both corresponding to the presence of the charge fluctuation) are at different sides of the dotted line. For the higher values of ϵ the mode is almost always unstable. This can be explained by recalling Eq. (6) where $\epsilon \equiv v_{id}/v_{ed} = 3$ implies that the fluctuation of the charge is mainly due to the collisions with ions; thus the electrons are already attached to the dust grains. In this situation eventual ion space charge variation cannot be efficiently screened by electrons. The resultant electric field is bigger and thus more free energy is available for the instability to grow. The increment therefore is bigger.

A similar effect of the electric field is presented in Fig. 6, where the critical values of the electric field E_c for the onset of the instability as a function of δ (the ratio of the negative charge on the dust and the total negative charge per unit volume) are given. We see that the greater the charge on the dust particles the smaller the number of electrons per unit volume, and the excitation of the DIA mode becomes easier, i.e., for smaller values of E_0 . The corresponding phase velocities are given in Fig. 7. The solid line in Fig. 6 is beneath the dotted one for the same reason as in Fig. 5, i.e., it is for



FIG. 7. The phase velocity $-\Omega_r/kc_{ia}$ versus δ corresponding to Fig. 6.



FIG. 9. Increment/decrement γ/kc_{ia} for the modes from Fig. 8. The dust (CF) mode is unstable for $\delta < 0.6$.

 ϵ =3, implying the lack of free electrons (that are attached to grains) which results in a more unstable system.

The mode behavior as a function of the negative charge on dust particles per unit volume δ is presented in Figs. 8 and 9. Here $k = 63 \text{ m}^{-1}$, $E_0 = 250 \text{ V/m}$, $P_c = 0.9$, and P =50 mtorr. The solid line in Fig. 8 represents the DIA frequency $-\Omega_r/kc_{ia}$ for $\epsilon = 3$, the dotted line is for the absence of charge fluctuations, and the dashed and dash-dotted lines are for $\epsilon = 0.3$. The dash-dotted line is the dust charge fluctuation mode, and it disappears for $\Omega_c = 0$. The corresponding growth rates γ/kc_{ia} are in Fig. 9. Only the CF mode (dash-dotted line) becomes stabilized by increasing the amount of dust in the system, i.e., decreasing the amount of electrons. This happens for approximately $\delta > 0.6$. The DIA mode becomes unstable for approximately $\delta > 0.8$, and δ >0.5 for $\epsilon = 0.3$ and $\Omega_c = 0$, respectively. One should note that the dashed and solid lines are at opposite sides of the dotted one.

IV. CONCLUSIONS

Clearly, an externally applied constant electric field can be used as an efficient generator of the ion-acoustic mode in a plasma containing electrons, ions, dust particles, and neutral gas. This has been known for a long time for the ordinary IA mode [12], and recently it has been discussed in detail for the mode in the presence of stationary dust particles [9].

However, our model suggests that charge fluctuation on the grain may also cause the excitation of the DIA mode. This feature suggests not only the possibility of investigating the DIA mode by using the CF mode as a probe, but also that the reverse can be done, i.e., the CF mode can be studied using the DIA mode. Therefore, charge fluctuation offers an interesting method of probing the high frequency dynamics of a dusty plasma.

We again emphasize the fact that the presence of the dust introduces two widely separated time scales into the system, i.e., the dust-acoustic time scale (of the order of hertz or tens of hertz), and the dust charge fluctuation scale (of the order of megahertz). The charge fluctuation frequency can become of the order of other plasma modes naturally appearing in such a system, like IA or DIA modes, and can severely modify their behavior. In Sec. II we discussed the effects of the dust charge fluctuations on the model investigated in Ref. [9]. We showed that the increment of the current-driven DIA mode is dependent on the dust charging frequency, having a maximum at some value of Ω_c . We demonstrated the presence of a dust charge fluctuation mode, which is unstable for dust charge concentrations not exceeding a certain given value.

The origin of the mode can be seen from the charge fluctuation equation (6) where charging (due to collisions) is proportional to the density of electrons and ions. In the presence of perturbations, i.e., the DIA mode (which physically represents a sequence of periodic compressions and rarefactions of the plasma), charging becomes space-time dependent, with a certain phase shift compared to the space-time dependent densities of electrons and ions, as can be clearly seen from Eq. (6), and it propagates as a mode.

In the regime when the charge fluctuation on dust particles is mainly due to the ion-dust collisions (the parameter ϵ =3) this CF mode strongly influences the behavior of the unstable DIA mode for wave numbers k in the interval approximately $10-100 \text{ m}^{-1}$. The effects of the intensity of the applied electric field E_0 were investigated and, as one might expect, the increment of the mode grows with increasing intensity of the driver. In the case when the electrons are depleted (ϵ =3) the mode is almost always unstable, and the increment is higher that usual. We also showed that the critical value of the driver E_0 decreases with increase of the relative charge on the dust particles, which one might expect since in that case the ion space charge variations are less efficiently screened by electrons. We believe that the present study should be useful in eventual experimental studies of the processes discussed in the text above.

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